

# Diffractive $\phi$ Production in a Perturbative QCD Model

J. Alam<sup>1</sup>, A. Hayashigaki<sup>2</sup>, K. Suzuki<sup>3</sup>, and T. Hatsuda<sup>4</sup>

*Department of Physics, University of Tokyo, Tokyo 113 0033, Japan*

## Abstract

The elastic lepto-production of  $\phi$  measured by the H1 collaboration at HERA is described by a perturbative QCD model, based on open  $s\bar{s}$  production and parton hadron duality, proposed by Martin et al. We observe that both the total cross section and the ratio of the longitudinal and transverse cross sections are well reproduced with an effective strange-quark mass  $m_s \sim 320 - 380$  MeV for various gluon distribution functions. Possible connection of the effective mass and the momentum dependent dynamical mass associated with dynamical breaking of chiral symmetry is also discussed.

PACS: 13.85.Ni,14.40.Gx,12.38.Bx,12.39.Jh

## I. Introduction

Recently the diffractive photo and lepto-production of vector mesons in electron proton collisions ( $e + p \rightarrow e + \gamma^* + p' \rightarrow e + V + p'$ ) has drawn considerable attention from experimental (see *e.g.* [1]) as well as theoretical side [2, 3, 4, 5, 6, 7, 8, 9]. As the cross section for the diffractive vector meson production depends (quadratically) on the gluon distribution of the proton, it gives a unique opportunity to study the low  $x$  behavior of the gluons inside the proton and to investigate the transition from perturbative to non-perturbative region. Experimental data for vector meson production

---

<sup>1</sup>E-mail : alam@nt.phys.s.u-tokyo.ac.jp

<sup>2</sup>E-mail : arata@nt.phys.s.u-tokyo.ac.jp

<sup>3</sup>E-mail : ksuzuki@nt.phys.s.u-tokyo.ac.jp

<sup>4</sup>E-mail : hatsuda@nt.phys.s.u-tokyo.ac.jp

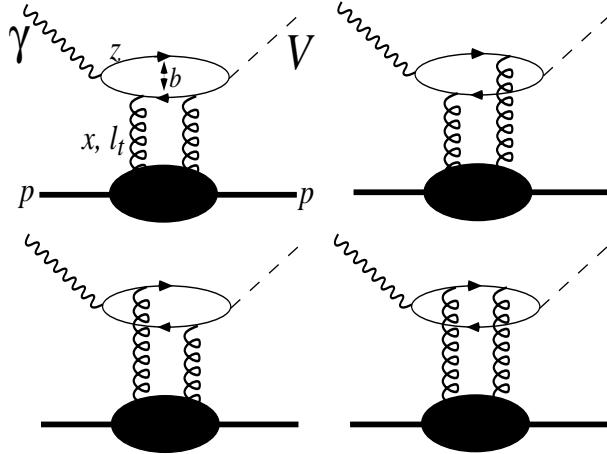


Figure 1: The diffractive vector meson production in  $ep$  collisions.  $b$  is the separation between the quark and the anti-quark.  $x$  and  $l_t$  ( $x'$  and  $-l_t$ , not shown in the figure) are the Bjorken- $x$  and transverse momentum of the left (right) gluon respectively.

at HERA [1, 10] in the reaction  $ep \rightarrow V ep$  are available over a wide range of the virtuality of the photon,  $Q^2$ , the *cm* energy of  $\gamma^* p$  system,  $W$ , mass of the vector mesons,  $m_V$  and the four momentum transfer,  $t$  in the process. The physical picture for the vector meson production is demonstrated through the diagrams in fig. 1. The virtual (or real) photon fluctuates to quark anti-quark ( $q\bar{q}$ ) pairs, which interact with the target proton via two gluons exchange. This interaction changes the transverse momenta of the pair, which subsequently hadronizes to a vector meson. It can be shown [11] that the time scale for the interaction of the  $q\bar{q}$  with the proton is considerably smaller than the time scale for the  $\gamma^* \rightarrow q\bar{q}$  dissociation and vector meson formation. This leads to the following factorization for the amplitude of diffractive vector meson production,

$$A(\gamma^* p \rightarrow V p) = \psi_{q\bar{q}}^{\gamma^*} \otimes A_{q\bar{q}+p} \otimes \psi_{q\bar{q}}^V, \quad (1)$$

where  $\psi_{q\bar{q}}^{\gamma^*}$  is the wave function of the virtual photon in  $q\bar{q}$ ,  $A_{q\bar{q}+p}$  is the amplitude for the  $q\bar{q}$ - $p$  interaction and  $\psi_{q\bar{q}}^V$  is the vector meson wave function.

The process under consideration is governed by the scale  $K^2 \sim z(1-z)(Q^2 + m_V^2)$ , indicating that for large  $Q^2$  and/or  $m_V^2$  the perturbative QCD

(pQCD) is applicable to describe the diffractive vector meson production [6, 12]. For the production of heavier mesons ( $J/\Psi$  or  $\Upsilon$ ) such process is under control by pQCD even for  $Q^2 = 0$  (photo-production). A reliable description for the heavy vector meson production is obtained by using eq. (1), which involves vector meson wave functions [4, 5, 6, 13]. These wave functions can be obtained by solving Schrödinger equation with non-relativistic potential model [14]. It has been argued in [15] that the main uncertainty to the description of the light vector mesons ( $\rho$ ) production (particularly in the transverse cross section) originates from its wave function. To avoid this problem, Martin et al. [15] proposed a model based on the open  $q\bar{q}$  production and parton-hadron duality to describe various features of  $\rho$  production in diffractive processes. Subsequently the same approach has been used to study the diffractive  $J/\Psi$  [16] and  $\Upsilon$  production [17].

Very recently the experimental data for elastic electroproduction of  $\phi$  mesons at HERA has been made available by the H1 collaboration [18]. In the present article we follow Ref. [17] to study the  $\phi$  production at HERA energies. The sensitivity of the results on the strange quark mass is examined. It is found in our analysis that the experimental data is described well with an effective strange quark mass, interpolated between the current and the constituent mass. In this approach one first calculates the amplitude for the open  $s\bar{s}$  production, then takes the projection of the amplitude on the  $J^P = 1^-$  state appropriate for  $\phi$  quantum numbers and finally integrating over an invariant mass interval such that it contains the resonance peak for the vector meson,  $\phi$ , here.

The paper is organized as follows. In the next section we discuss the model used in the present work. Section III is devoted to present the results and finally in section IV we give summary and conclusions.

## II. pQCD Model

The differential cross section for the open  $s\bar{s}$  production from a longitudinally ( $L$ ) or transversely ( $T$ ) polarized photon can be written as [16, 19],

$$\frac{d\sigma^{L(T)}}{dM^2 dt} = \frac{2\pi^2 e_s^2 \alpha}{3(Q^2 + M^2)^2} \int dz \sum_{i,j} |B_{ij}^{L(T)}|^2, \quad (2)$$

where  $e_s$  is the charge of the strange quark,  $\alpha$  is the fine structure constant,  $M$  is the invariant mass of the  $s\bar{s}$  pair,  $z(1-z)$  is the light cone fraction of the photon momentum carried by the quark (anti-quark) and  $B_{ij}^{L(T)}$  is the helicity amplitude for the dissociation of a  $L(T)$  polarized photon into a  $s\bar{s}$  pair with helicities  $i$  and  $j$  respectively. For transversely and longitudinally polarized photon the amplitudes (for  $t = 0$ ) are given by [16],

$$\begin{aligned} \text{Im}B_{++}^T &= \frac{m_s I_L}{2h(z)}, \quad \text{Im}B_{+-}^T = \frac{-zk_T I_T}{h(z)}, \\ \text{Im}B_{-+}^T &= \frac{(1-z)k_T I_T}{h(z)}, \quad B_{--}^T = 0, \\ \text{Im}B_{+-}^L &= -\text{Im}B_{-+}^L = \sqrt{\frac{Q^2}{2}}h(z)I_L, \quad B_{++}^L = B_{--}^L = 0, \end{aligned} \quad (3)$$

where  $h(z) = \sqrt{z(1-z)}$  and

$$I_L = K^2 \int^{K^2} \frac{dl_t^2}{l_t^4} \alpha_s(l_t^2) f(x, x', l_t^2) \left( \frac{1}{K^2} - \frac{1}{K_l^2} \right), \quad (4)$$

$$I_T = \frac{K^2}{2} \int^{K^2} \frac{dl_t^2}{l_t^4} \alpha_s(l_t^2) f(x, x', l_t^2) \left( \frac{1}{K^2} - \frac{1}{2k_T^2} + \frac{K^2 - 2k_T^2 + l_t^2}{2k_T^2 K_l^2} \right). \quad (5)$$

$k_T$  ( $-k_T$ ) is the transverse momentum of the quark (anti-quark),  $K^2 = z(1-z)Q^2 + k_T^2 + m_s^2$ , is the scale probed by the process and  $K_l^2 = \sqrt{(K^2 + l_t^2)^2 - 4k_T^2 l_t^2}$ .  $f(x, x', l_t^2)$  is the skewed (off-diagonal) gluon distribution un-integrated over its transverse momentum,  $l_t$ .  $x \simeq (Q^2 + M^2)/(W^2 + Q^2)$  and  $x' \simeq (M^2 - m_V^2)/(W^2 + Q^2) (\ll x)$ , which indicates that heavier the mesons more important is the skewness. In the present article we use diagonal gluon distribution,  $g(x, l_t^2)$ , related to  $f(x, l_t^2)$  as follows,

$$f(x, l_t^2) = \frac{\partial(xg(x, l_t^2))}{\partial \ln l_t^2}. \quad (6)$$

The skewness of the gluon distribution has been taken into account by multiplying the amplitudes by a factor  $R_g$  [16],

$$R_g = \frac{2^{2\lambda+3} \Gamma(\lambda + \frac{5}{2})}{\sqrt{\pi} \Gamma(\lambda + 4)}, \quad (7)$$

where  $\lambda \sim \partial \ln(xg(x, Q^2)) / \partial \ln(1/x)$ .

The contribution from the infrared region has been obtained by introducing an infrared separation scale,  $l_0^2$  [19].

$$I_L = \alpha_s(l_0^2) x g(x, l_0^2) \left( \frac{1}{K^2} - \frac{2k_T^2}{K^4} \right) + K^2 \int_{l_0^2}^{K^2} \frac{dl_t^2}{l_t^4} \alpha_s(l_t^2) f(x, x', l_t^2) \left( \frac{1}{K^2} - \frac{1}{K_l^2} \right), \quad (8)$$

and similarly

$$\begin{aligned} I_T = & \alpha_s(l_0^2) x g(x, l_0^2) \left( \frac{1}{K^2} - \frac{k_T^2}{K^4} \right) + \frac{K^2}{2} \int_{l_0^2}^{K^2} \frac{dl_t^2}{l_t^4} \alpha_s(l_t^2) f(x, x', l_t^2) \\ & \times \left( \frac{1}{K^2} - \frac{1}{2k_T^2} + \frac{K^2 - 2k_T^2 + l_t^2}{2k_T^2 K_l^2} \right). \end{aligned} \quad (9)$$

The amplitudes,  $B_{ij}^{L(T)}$ , given above are evaluated in the proton rest frame. As the formation of the  $\phi$  takes place in the rest frame of the  $s\bar{s}$ , it is required to transform the helicity amplitude from the proton rest frame to the  $s\bar{s}$  rest frame through the transformation,

$$A_{kl} = \sum_{i,j} c_{ik} c_{lj} B_{ij}, \quad (10)$$

where

$$c_{++} = c_{--} = c_{+-} = -c_{-+} = \sqrt{(1 - a \cdot b)/2}, \quad (11)$$

$a_\mu$  is the quark polarization vector in the  $s\bar{s}$  rest frame and  $b_\mu$  is the corresponding quantities in the proton rest frame [17].

Having obtained these values for the amplitudes in the  $s\bar{s}$  rest frame we take the projections of these amplitudes in the  $J^P = 1^-$  states by the following equation,

$$A_{jk}^{L(T)} = \sum_J e_J^{L(T)} d_{1\beta}^J, \quad (12)$$

where  $d_{1\beta}^J$  are the spin rotation matrices [20] and  $e_J^{L(T)}$  could be obtained by inverting the above relation.

The amplitudes given in eqs. (3) contain only the imaginary part, while the real parts are obtained by using the relation  $\text{Re } A = \tan(\pi\lambda/2) \text{Im } A$  [17]. In the present work the NLO correction has been taken into account by multiplying the amplitudes by a  $K$  factor,  $K = \exp(\pi C_F \alpha_s / 2)$ , where the scale used as the argument of  $\alpha_s$  is  $2K^2$  and  $C_F = 4/3$ .

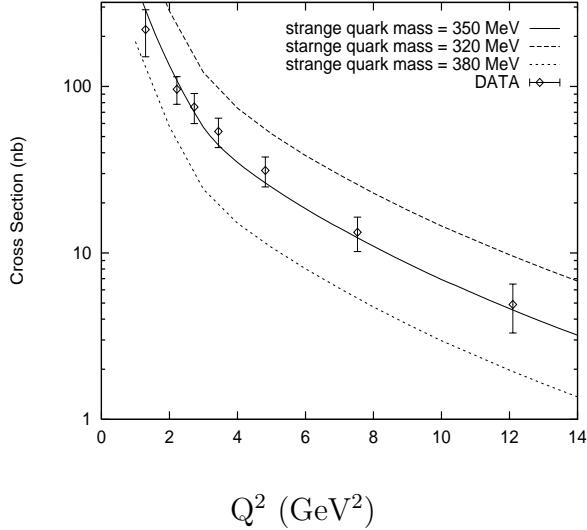


Figure 2: Diffractive  $\phi$  production cross section as a function of  $Q^2$  at HERA for GRV98(NLO) gluon distribution and for three values of the strange quark masses,  $m_s = 320, 350$  and  $380$  MeV. The  $cm$  energy of the  $\gamma^* p$  system is  $W = 75$  GeV and  $l_0^2 = 1$  GeV $^2$ .

### III. Results

The cross section for the  $\phi$  production from longitudinally and transversely polarized photon are obtained by integrating eq. (2) (with the amplitudes projected in the  $J^P = 1^-$  state) over an mass interval  $1.0 \text{ GeV} \leq M \leq 1.04 \text{ GeV}$ , as the  $\phi$  has been experimentally observed in this invariant mass interval through  $\phi \rightarrow K \bar{K}$  decay [18]. The  $t$  integration has been performed by assuming a  $t$  dependence of the cross section  $\sim \exp(bt)$ , with an average slope,  $b = 5.2 \text{ GeV}^{-2}$ , taken from experiment [18].

The typical value of  $x$  sampled in diffractive  $\phi$  production for  $W = 75$  GeV is  $x \sim 2 \times 10^{-4}$ . For such a low value of  $x$  there is a large ambiguity among various parametrizations of the gluon distribution [21, 22, 23]. Therefore, we will show the sensitivity of our results on the gluon distributions. We start with the GRV98(NLO) gluon distribution. In fig. 2, the  $Q^2$  dependence of the cross section is depicted. For the strange quark mass,  $m_s = 350$  MeV, and the infrared scale,  $l_0^2 = 1$  GeV $^2$ , the agreement between the QCD based

description and the experimental data is reasonably good. The sensitivity of the cross section on the strange quark mass is evident from the figure. The data can be fitted by appropriately increasing (decreasing) the invariant mass interval for  $m_s = 380(320)$  MeV, but in the present work we prefer to fix the window in the range  $1 \leq M \leq 1.04$  GeV because of the reason mentioned earlier. With current quark mass,  $m_s \sim 150$  MeV, it is observed that the theoretical results overestimate the data by a large amount with the above invariant mass window.

We note that the experimental data is well reproduced with the strange quark mass,  $m_s \sim 350$  MeV which is intermediate between the constituent mass ( $M_s \sim 500$  MeV) and the current mass ( $m_{s,0} \simeq 150$  MeV). At this point we recall the momentum-dependent effective strange-quark mass  $m_s(p)$ , which interpolates between the constituent mass and the current mass, may be realized through the dynamical breaking of chiral symmetry [24, 25, 26]. In particular, for large space-like momentum  $p^2 = -P^2 < 0$  ( $P^2$ : large), the operator product expansion for the quark propagator yields the asymptotic behavior [25]

$$m_s(P) = m_{s,0}(\mu) \left( \frac{\alpha_s(P)}{\alpha_s(\mu)} \right)^d + \frac{16\pi\alpha_s(P)}{P^2} |\langle \bar{\psi}\psi(\mu) \rangle| \left( \frac{\alpha_s(P)}{\alpha_s(\mu)} \right)^{-d}, \quad (13)$$

where  $d (= 12/27$  for  $N_f = 3$ ) is the mass anomalous dimension,  $\mu$  is the renormalization point, and  $\langle \bar{\psi}\psi(\mu) \rangle$  is the chiral vacuum condensate.

In Fig. 3, the asymptotic form of  $m_s(P)$  as a function of the space-like momentum  $P$  is shown, where  $m_{s,0}(2\text{GeV})=118.9 \pm 12.2$  MeV,  $m_{u,0}(2\text{GeV})=3.5 \pm 0.4$  MeV,  $m_{d,0}(2\text{GeV})=6.3 \pm 0.8$  and  $m_\pi^2 f_\pi^2 \simeq -(m_u + m_d) \langle \bar{\psi}\psi \rangle$  are taken from the first Ref. of [27]. To obtain  $m_s(P)$  for entire domain of space-like and time-like momenta, one needs to solve the Schwinger-Dyson equation for quark propagator with suitable assumption on the gluon propagator and the quark-gluon vertex at low energies (see, e.g., [26, 28, 29]). In that case,  $m_s(P)$  is expected to be a smooth interpolation between the asymptotic behavior eq.(13) at large  $P^2$  and the constituent mass,  $M_s \simeq 500$  MeV at  $P \sim 0$ . In our diffractive process, we need  $m_s(P)$  in the entire domain of  $P$  in principle, since the quarks with space-like momentum are initially produced by the space-like photon ( $Q^2 < 0$ ) and they eventually become time-like after the *kick* by the gluons inside the proton (see Fig.1). The quark mass, we have found in our analysis,  $m_s \sim 350$  MeV, which is smaller than  $M_s$  but is

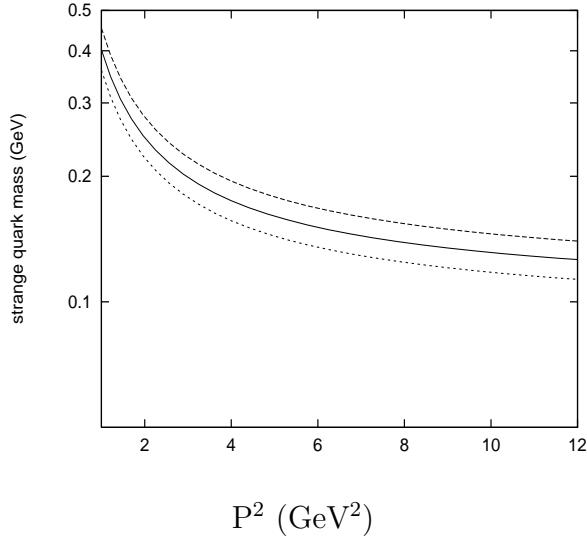


Figure 3: The variation of the strange quark mass as a function of  $P^2$  taking into account the uncertainty of the current masses,  $m_{u,0}(2\text{GeV})= 3.5 \pm 0.4$  MeV,  $m_{d,0}(2\text{GeV})= 6.3 \pm 0.8$  MeV and  $m_{s,0}(2\text{GeV})= 118.9 \pm 12.2$  MeV (see text). These current masses are taken from Ref. [27].

larger than  $m_{s,0}$ , may be thus interpreted as an effective mass averaged over momentum relevant for the diffractive process shown in Fig.1.

In fig.4 we show the ratio  $R = \sigma_L/\sigma_T$  as a function of  $Q^2$ . The theoretical calculation shows the correct trend. Putting the constraint on the strange quark mass and the infrared scale from the experimental data shown in fig.2, we evaluate the  $W$  dependence of the cross section for various values of  $Q^2$ . The agreement between the experimental data (taken from [1]) and the theoretical calculation is satisfactory. In fig.6 we show the results for various values of  $l_0^2$  with  $m_s = 350$  MeV. Results obtained for  $l_0^2 = 1.5$  and  $2 \text{ GeV}^2$  start deviating from the experimental value at small  $Q^2$ .

In fig.7 we show the  $Q^2$  behavior of the cross section for different gluon distribution functions. The value of the infrared scale,  $l_0^2$  and strange quark mass,  $m_s$  are  $1.5 \text{ GeV}^2$  and  $320$  MeV respectively. Although GRV98(NLO) gluon distribution overestimates, the predictions with MRST99 and CTEQ5M gluon distributions are in agreement with the experimental results. With smaller values of  $m_s \sim 150$  MeV, however, we fail to describe the data

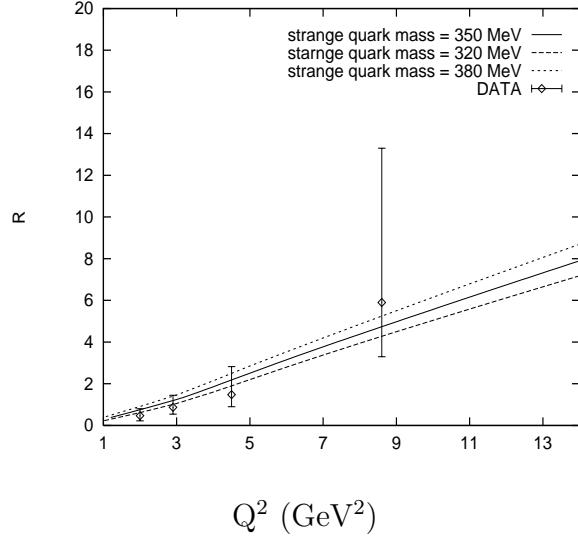


Figure 4: The ratio  $R = \sigma_L / \sigma_T$  as a function of  $Q^2$  for GRV98(NLO) gluon distribution for  $l_0^2 = 1 \text{ GeV}^2$ .

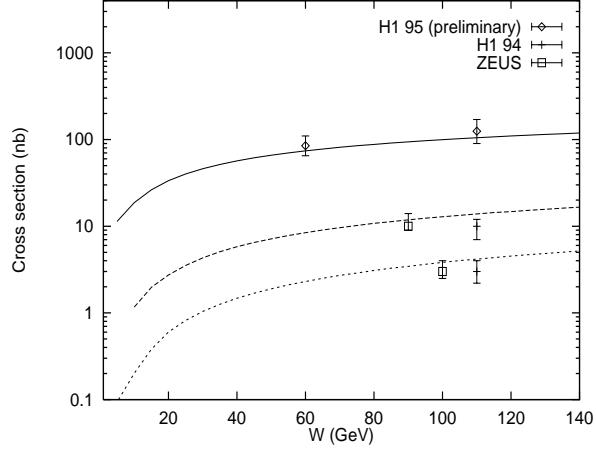


Figure 5: The cross section for  $\phi$  production as a function of  $W$  for  $Q^2 = 2.5$  (solid line),  $8.3$  (dashed line) and  $14.6 \text{ GeV}^2$  (dotted line) for GRV98(NLO) gluon distribution with  $l_0^2 = 1 \text{ GeV}^2$  and  $m_s = 350 \text{ MeV}$ .

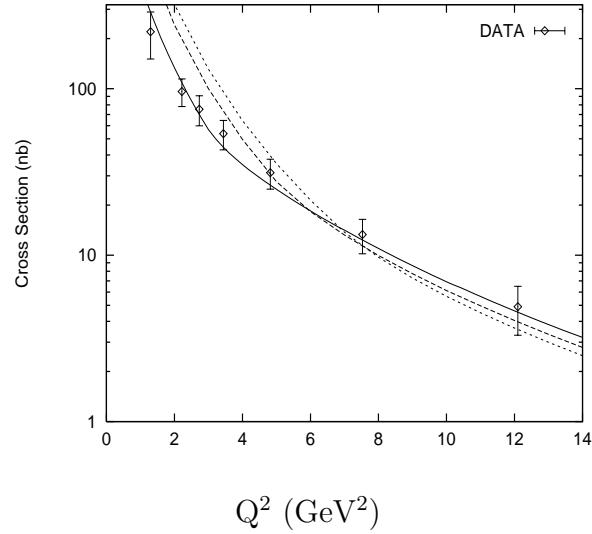


Figure 6: The cross section for  $\phi$  production as a function of  $Q^2$  for  $W = 75$  GeV for GRV98 (NLO) gluon distribution for  $l_0^2 = 1$  (solid line), 1.5 (dashed line), and  $2 \text{ GeV}^2$  (dotted line) with  $m_s = 350$  MeV.

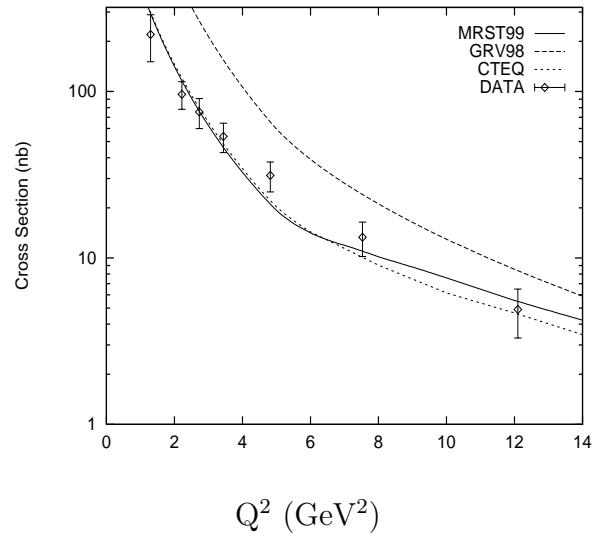


Figure 7: Diffractive production of  $\phi$  at HERA for various parametrization of gluon distribution function.  $m_s = 320$  MeV and  $l_0^2 = 1.5 \text{ GeV}^2$  are adopted.

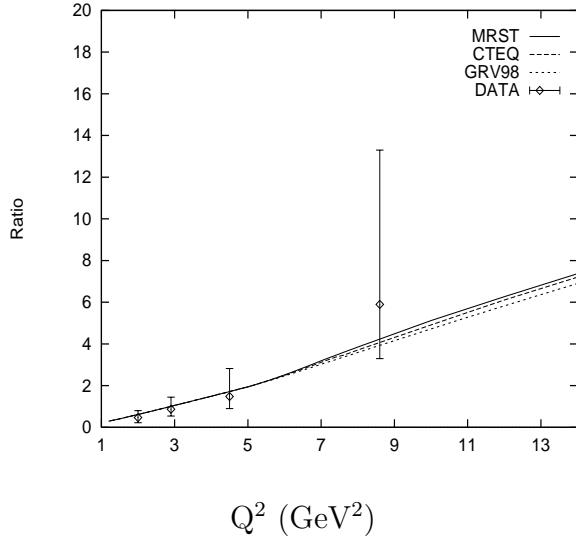


Figure 8: The ratio  $R = \sigma_L/\sigma_T$  as a function of  $Q^2$  for various parametrization of the gluon distribution function with  $m_s = 320$  MeV and  $l_0^2 = 1.5$   $\text{GeV}^2$ .

provided the invariant mass window is kept fixed at  $1 \leq M \leq 1.04$   $\text{GeV}$ , where the  $\phi$  has been measured experimentally. Fig. 8 indicates the variation of  $\sigma_L/\sigma_T$  as function of  $Q^2$  for CTEQ5M, GRV98(NLO) and MRST gluon distributions, the ratio seems to be less sensitive to the gluon distributions.

## IV. Summary and Conclusions

We have studied the diffractive  $\phi$  production at HERA energies measured by H1 collaboration within the ambit of pQCD model based on parton hadron duality. The effects of the off-diagonal gluon distribution and the NLO corrections through the  $K$ -factor have been incorporated. The sensitivity of the results on the infrared separation scale and the various parametrization of the gluon distribution have been discussed. It is found that, with reasonable choice of the infrared scale, the total cross section and the ratio,  $\sigma_L/\sigma_T$  are well described in the present framework with an effective strange quark mass,  $m_s \sim 320 - 380$  MeV. Such a value of the strange quark mass, which

lies between constituent and current quark masses may be closely related to the momentum-dependent dynamical mass associated with the dynamical breaking of chiral symmetry in QCD.

**Acknowledgement:** We are grateful to K. Itakura for useful discussions. K.S. and A. H. are supported by JSPS Research Fellowship for Young Scientists. J.A. is grateful to the Japan Society for Promotion of Science (JSPS) for financial support. J.A. and T.H. are also supported by Grant-in-aid for Scientific Research No. 98360 of JSPS.

## References

- [1] H. Abramowicz and A. Caldwell, Rev. Mod. Phys. **71** (1999) 1275.
- [2] A. Hebecker, Phys. Rep. **331** (2000) 1.
- [3] A. Donnachie and P. V. Landshoff, Phys. Lett. **B 185** (1987) 403; erratum - *ibid*, **B 198** (1987) 590; Nucl. Phys. **B 311** (1989) 509.
- [4] B. Z. Kopeliovich, J. Nemchick, N. N. Nikolaev, and B. G. Zakharov, Phys. Lett. **B 309** (1993) 179.
- [5] J. Nemchick, N. N. Nikolaev, E. Predazzi and B. G. Zakharov, Z. Phys. **C 75** (1997) 71.
- [6] S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Mueller and M. Strikman, Phys. Rev. **D 50** (1994) 3134.
- [7] I. Royen and J. R. Cudell, Nucl. Phys. **B 545** (1999) 505.
- [8] K. Suzuki, A. Hayashigaki, K. Itakura, J. Alam and T. Hatsuda, Phys. Rev. D (in press).
- [9] P. Hoyer and S. Peigne, Phys. Rev. **D 61** (2000) R031501.
- [10] H1 Collaboration, C. Adloff et al. Eur. Phys. J. **C 10** (1999) 373.
- [11] M. Wüsthoff and A. D. Martin, J. Phys. **G 25** (1999) R309.

- [12] J. C. Collins, L. Frankfurt, M. Strikman, Phys. Rev. **D56** (1997) 2982.
- [13] L. Frankfurt, W. Koepf and M. Strikman, Phys. Rev. **D57** (1998) 512.
- [14] E. J. Eichten and C. Quigg, Phys. Rev. **D 52** (1995) 1726.
- [15] A. D. Martin, M. G. Ryskin and T. Teubner, Phys. Rev. **D55** (1997) 4329.
- [16] A. D. Martin, M. G. Ryskin and T. Teubner, Phys. Rev. **D 62** (2000) 014022.
- [17] A. D. Martin, M. G. Ryskin and T. Teubner, Phys. Lett. **B454** (1999) 339.
- [18] H1 Collaboration, C. Adloff et al., hep-ex/0005010.
- [19] E. M. Levin, A. D. Martin, M. G. Ryskin and T. Teubner, Z. Phys. **C 74** (1997) 671.
- [20] A. R. Edmonds, *Angular Momentum in Quantum Mechanics*, Princeton University Press, Princeton, 1957.
- [21] H. L. Lai et al., Eur. Phys. J. **C 12** (2000) 375.
- [22] M. Glück, E. Reya and A. Vogt, Eur. Phys. J. **C 5** (1998) 461.
- [23] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. **C 14** (2000) 133.
- [24] H. Georgi and H. D. Politzer, Phys. Rev. **D 14** (1976) 1829.
- [25] H. D. Politzer, Nucl. Phys. **B 117** (1976) 397.
- [26] K. Higashijima, Phys. Rev. **D 29** (1984) 1228; Prog. Theor. Phys. Suppl. **87** (1992) 441.
- [27] S. Narison, hep-ph/9911454; J. Gasser and H. Leutwyler, Phys. Rep. **87** (1982) 77.
- [28] R. Fukuda and T. Kugo, Nucl. Phys. **B117** (1976) 250; A. Bender and R. Alkofer, Phys. Rev. **D 53** (1996) 446.
- [29] C. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477.